a Comparative Study

J.C.Giacomin*, P.C.Cortizo**, P.F.D.Garcia**, W.S.Lacerda*

* Departamento de Ciências Exatas Universidade Federal de Lavras 37200 – 000 – Lavras – MG - Brasil e-mail: giacomin@ufla.br Fax: (035) 829-1371

Abstract - This paper presents a study on two single-phase switching AC/DC converters used to supply energy to a resistive variable load with constant voltage output, obtaining High Power Factor input with Low Harmonic Distortion. The dynamic and Steady-State behavior of two DC power suplies is analysed by their State-Equations and Transfer Functions. The Capacitive Idling Cuk Converter [1] [2] which is used to obtain both High Power Factor input, and constant voltage output, is studied, as well as the commonly used pre-regulator Boost and regulator Buck (named Boost-Buck set), coupled, operating in continuous current conduction mode (CCM). Low power prototypes were built using the same components in both converters, and practical results were obtained to compare them. A isolated Boost-Buck set was built, using a Forward converter in place of the Buck converter.

I. INTRODUCTION

Recent researchs in AC/DC converters have been focused in obtaining high power factor (HPF) input with low current harmonic distortion, especially to attend International Standards as IEC-61000-3-2 [3]. Although these efforts have been great and rewarding, very little has been done in associating this high quality input energy with high quality output voltage. In some cases, in order to achieve one feature, the other is penalized. In other cases, good results are obtained in both high quality input energy and output voltage, but the dynamic behavior is not so good under load variation. Therefore, it is necessary to investigate DC power supplies which operate with HPF and are useful in supplying energy to variable loads.

This work is dedicated to study the Capacitive Idling Cuk Converter and the Boost-Buck set (preregulator Boost and regulator Buck). Both of them operate in CCM (Continuous current Conduction Mode) and Hard Switching mode. This operation mode was chosen because (a) it is simpler to control and (b) it presents efficiency and performance as good as other alternative topologies (one stage converters, soft-switching converters) [4] [5]. A set of dynamic algebraic equations is developed and a linear model for the converters is built, employing State Space Averaging, which points to a dynamic coupling between input and output stages. The Converter project and the ** CPDEE
Universidade Federal de Minas Gerais
31270-901 – Belo Horizonte – MG – Brasil
e-mail: porfirio@cpdee.ufmg.br
Fax: (031)499-5480

study of the Transfer Functions were made, as well as the project of controlers to be used.

The algebraic study and the simulations indicated equality in performance for both converters, whereas practical implementation and results pointed some differences.

A electrically isolated version of Boost-Buck set was studied, replacing the Buck by a Forward converter, and appling the results of the studies and control project to the new set. For the Boost-Forward set was used a 13.8V output voltage, whereas for Boost-Buck set was used a 69V output voltage (5*13.8V).

II. THE CONVERTERS

The Capacitive Idling Cuk converter is derived from basic Cuk converter, which uses an aditional switch in order to control the energy sent to the output [1] [2]. Its squematic diagram is presented in fig. 1.b. This aditional switch permits an independency between the three main voltages in the Cuk Converter (in Cuk converter V_1 =Vi+Vo). Switch S₁ controls the energy absorved from the input AC line and stored in capacitor C₁, whereas S₂ controls the energy transferred from C₁ to the output. Then C₁ functions as a bulk capacitor, absorving the intrinsic low frequency oscilations from the input energy. Operating S₂, it is possible to quickly control the output voltage, which is vital for a switching power supply.

Figure 1.a shows the known Boost-Buck set diagram. Although Capacitive Idling Cuk converter has constructive and behavioral similarities with Boost-Buck set, it is necessary to impose some restrictions in its operation: switch S_1 must remain closed while S_2 is closed $(D_1 \ge D_2)$; and the main voltages must obey the inequality $V_1 \ge Vi+Vo$. These restrictions may be imposed by the transistors command circuitry or may be achieved naturaly by the control circuitry, when the control constants are chosen properly. In this work, the condition $D_1 \ge D_2$ arised naturaly.

In Stead-State the condition $V_1 \ge Vi + Vo$ is a consequence of $D_1 \ge D_2$, but this is not true for the Transitory State. Then both conditions must be observed when Capacitive Idling Cuk converter is projected.



Fig 1 - Converters: (a) Boost-Buck set; (b) Capacitive Idling Cuk converter

The isolated version for Boost-Buck set is obtained replacing Buck by a Forward converter, and in Capacitive Idling Cuk converter it is obtained by adding a transformer close to capacitor C_1 , as in standard Cuk converter. In this work is not demonstrated the possibility of applying the control projects results of the converters to their isolated versions. Only practical results were obtained to the Boost-Foward set.

III. THE MODELING

Figure 1.a presents the converters Boost and Buck cascaded (coupled in series), using a simplified model, where the inductors' resistencies and equivalent series resistancies (ESR) of the capacitors are showed, and transistors and diodes are assumed to be ideal switches. Their dynamic equations (1) are obtained by State Space Averaging [6][7]. Transistor 1 (Boost) is represented by S_1 and transistor 2 (Buck) is represented by S_2 .

Using State Space Averaging, the dynamic behavior of the converter (Boost-Buck set) is described by its State-Equations:

$$\mathbf{\dot{X}} = \mathbf{AX} + \mathbf{BU}$$
$$\mathbf{Y} = \mathbf{CX} + \mathbf{DU}$$
(1)

Matrix A, B, D, X and U are:

$$\mathbf{A} = \begin{bmatrix} -\frac{r_a}{L_1} & -\frac{(1-D_1)}{L_1} & \frac{D_3 r_3}{L_1} & 0\\ \frac{(1-D_1)}{C_1} & 0 & -\frac{D_2}{C_1} & 0\\ \frac{D_3 r_3}{L_2} & \frac{D_2}{L_2} & -\frac{r_b}{L_2} & -\frac{R}{L_2(r_4+R)}\\ 0 & 0 & \frac{R}{C_2(r_4+R)} & -\frac{1}{C_2(r_4+R)} \end{bmatrix}$$
(2)

$$\mathbf{B} = \begin{bmatrix} 1/L_1 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$
(3)

$$\mathbf{D} = \begin{bmatrix} 0 \end{bmatrix} \tag{4}$$

$$\mathbf{X} = \begin{bmatrix} I_1 & V_1 & I_2 & V_2 \end{bmatrix}^T$$
(5)

$$\mathbf{U} = \begin{bmatrix} Vi \end{bmatrix} \tag{6}$$

where:

$$r_a = r_1 + (1 - D_1)r_3 \qquad , \tag{7}$$

$$r_b = r_2 + D_2 r_3 + \frac{r_4 \times R}{r_4 + R}$$
, (8)

and superscript T denotes matrix transposition.

 D_1 is the duty ratio of Boost converter, and D_2 is the duty ratio of Buck converter. D_3 is determined by the period of time in which S_1 is opened and S_2 is closed. These equations are the same for the Capacitive Idling Cuk Converter, except for the terms with D_3 .

The matrix C depends on the observed State-Variable. For example, if voltage V_2 is desired:

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

The 4 State-Variables Transfer Functions (I₁, V₁, related to D₁; and I₂, V₂ related to D₂) developed from the State-Equations of the converters were very complex. A numeric study made using real values of the components used to built the converters pointed out to the possibility to regard null the equivalent series resistancies (ESR) of the capacitors ($r_3=r_4=0$). Furthermore, it is reasonable to think about neglecting these resistancies since they are the same order the intrinsic resistencies of transistor and diodes. This simplifications allows the derivation of appropriate Transfer Functions for the study of coupled Boost and Buck converters.

The DC Transfer Function (for large signals) between input voltage and output voltage, for both converters (Boost-Buck set and Capacitive Idling Cuk converter) is given by (10):

$$\frac{V_o}{V_i} = -\mathbf{C}\mathbf{A}^{-1}\mathbf{B} = \frac{(1-D_1)D_2R}{(1-D_1)^2(r_2+R) + D_2^2r_1}$$
(10)

One can say that in this type of converter, the input and the output will be decoupled if the function which describes the output voltage can be made as a product of two functions [1]:

$$Vo = f_1(Vi, D_1) * f_2(I_2, D_2)$$
(11)

It can easely be seen that (10) matches (11), if $r_1 << R$ and $r_2 << R$ what indicates the possibility of decoupling for large signals between the two converters. We must verify the possibility of dynamic decoupling (i.e., small signals) for the converters.

IV. DYNAMIC DECOUPLING

The equations describing the small signal Transfer Functions of voltage and current for the preregulator related to D_1 (duty ratio of S_1), and voltage and current for the regulator related to D_2 (duty ratio of S_2), are of the types indicated in (12), (13), (14), (15),. Lowercase letters d, v, i, represent small perturbations in variables, D, V, I, respectively.

$$G_{I1}(s) = \frac{i_1(s)}{d_1(s)} = \frac{(1-D_1)V_i}{L_1} \frac{(R+r_2)}{R_{eq}} \frac{s^3 + c_2 s^2 + c_1 s + c_0}{\Delta(s)}$$
(12)

$$G_{V1}(s) = \frac{v_1(s)}{d_1(s)} = \frac{V_i}{R_{eq}} \frac{D_2^2}{C_1} \frac{-s^3 + b_2 s^2 + b_1 s + b_0}{\Delta(s)}$$
(13)

$$G_{I2}(s) = \frac{i_2(s)}{d_2(s)} = \frac{(1-D_1)V_i}{L_2} \frac{(R+r_2)}{R_{eq}} \frac{s^3 + h_2 s^2 + h_1 s + h_0}{\Delta(s)}$$
(14)

$$G_{V2}(s) = \frac{v_2(s)}{d_2(s)} = \frac{(1 - D_1)V_i}{L_2C_2} \frac{(R + r_2)}{R_{eq}} \frac{s^2 + f_1s + f_0}{\Delta(s)} \quad (15)$$

where:

$$R_{eq} = (1 - D_1)^2 (R + r_2) + D_2^2 r_1 \quad , \tag{16}$$

and the characteristic polynomial common to the previous four equations is of the type:

$$\Delta(s) = s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 .$$
 (17)

The Transfer Functions indicated above are very complex and indicate a fourth order dynamic behavior for the converters. A more simplified study is obtained when it is made null the resistancies of inductors. Then the characteristic poliniomial will be described as (18):

$$\Delta(s) = s^{4} + \frac{1}{C_{2}R}s^{3} + \left[\frac{1}{C_{2}L_{2}} + \frac{(1-D_{1})^{2}}{C_{1}L_{1}} + \frac{D_{2}^{2}}{C_{1}L_{2}}\right]s^{2} + \frac{1}{C_{1}C_{2}R}\left[\frac{(1-D_{1})^{2}}{L_{1}} + \frac{D_{2}^{2}}{L_{2}}\right]s + \frac{(1-D_{1})^{2}}{C_{1}L_{1}C_{2}L_{2}}$$
(18)

One factor is important to achieve dynamic decoupling between the input and the output converters, namely the amount of energy stored in capacitor C_1 (Bulk capacitor). Then the greater is the capacitance of C_1 and the higher is its voltage, the better will be the decoupling.

In practice the characteristic polinomial might be approximated by the product of two second order

polinomials (19), as long as the ratio D_2^2/C_1 is small enough compared to the other terms of the equation.

$$\Delta(s) = P_1(s) \bullet P_2(s) \tag{19}$$

where $P_1(s)$ and $P_2(s)$ are:

$$P_1(s) = \left[s^2 + \frac{D_2^2}{RC_1}s + \frac{(1-D_1)^2}{C_1L_1}\right]$$
(20)

$$P_2(s) = \left[s^2 + \frac{1}{RC_2} + \frac{1}{C_2L_2}\right]$$
(21)

 $P_2(s)$ is the characteristic polynomial of a Buck converter, and $P_1(s)$ approximates the Boost converter's characteristic polynomial.

Appling the same approximations, it is possible to describe the Transfer Functions as:

$$G_{I1}(s) = \frac{Vi}{(1-D_1)L_1} \frac{\left(s + \frac{2D_2^2}{RC_1}\right)P_2(s)}{\Delta(s)}$$
(22)

$$G_{V1}(s) = \frac{Vi}{(1-D_1)^2 C_1} \frac{D_2^2}{R} \frac{\left(-s + \frac{R}{D_2^2} \frac{(1-D_1)^2}{L_1}\right) P_2(s)}{\Delta(s)}$$
(23)

$$G_{I2}(s) = \frac{Vi}{(1-D_1)L_2} \frac{\left(s + \frac{1}{RC_2}\right)P_3(s)}{\Delta(s)}$$
(24)

$$G_{V2}(s) = \frac{Vi}{(1 - D_1)L_2C_2} \frac{P_3(s)}{\Delta(s)}$$
(25)

where:

$$P_3(s) = s^2 - \frac{D_2^2}{RC_1}s + \frac{(1 - D_1)^2}{L_1C_1}$$
(26)

In [1] is demonstrated that $P_3(s)$ can cancel out with $P_1(s)$ when the inductors' resistancies $(r_1 \text{ and } r_2)$ are considered in the equations. The first order coefficients of $P_1(s)$ and $P_3(s)$ were:

$$p_{11} = \frac{r_1}{L_1} + \frac{D_2^2}{C_1(R+r_2)}$$
(27)

and

$$p_{31} = \frac{r_1}{L_1} - \frac{D_2^2}{C_1(R + r_2)}$$
(28)

In this way, the Transfer Functions would be:

$$G_{I1}(s) = \frac{Vi}{(1-D_1)L_1} \frac{\left(s + \frac{2D_2^2}{RC_1}\right)}{P_1(s)}$$
(29)

$$G_{V1}(s) = \frac{Vi}{(1-D_1)^2 C_1} \frac{D_2^2}{R} \frac{\left(-s + \frac{R}{D_2^2} \frac{(1-D_1)^2}{L_1}\right)}{P_1(s)}$$
(30)

$$G_{I2}(s) = \frac{Vi}{(1 - D_1)L_2} \frac{\left(s + \frac{1}{RC_2}\right)}{P_2(s)}$$
(31)

$$G_{V2}(s) = \frac{Vi}{(1-D_1)L_2C_2} \frac{1}{P_2(s)}$$
(32)

One can verify that (29) and (30) correspond to the Boost converter Transfer Functions, since the resistance *R* at Buck output represents, for the Boost converter, a load described by (33):

$$R_{Boost} = \frac{R}{D_2^2} \,. \tag{33}$$

One can also verify that (31) and (32) correspond to Buck Transfer Functions, as long as the input voltage for Buck is:

$$Vi_{Buck} = \frac{Vi}{1 - D_1} \,. \tag{34}$$

Then, it was demonstrated the dynamic independence, although imperfect, between two coupled converters, employed as pre-regulator and output voltage regulator.

A numeric study for real converters was made, using Bode plots for the fourth order and simplified second order Transfer Funcitions. The results have showed a nearly perfect soperposition between the diagrams correspondents.

V. THE CONVERTERS

A project was made for a low power prototype (<150W) in order to attend the specifications below:

line voltage (Boost):	$V_l = 127 {\rm Vrms} / 60 {\rm Hz}$		
output voltage (Buck):	$V_o = 69 \mathrm{V}$		
output current:	$I_o = 0.5 \sim 2.0 \text{A}$		
switching frequency:	Fs = 50 kHz		

The pre-regulator (Boost) and regulator (Buck) inductors operating in CCM are calculated as (35) and (36) respectively [6][7][8][9]:

$$L_1 = \frac{0.32 V_{ip}}{\Delta I_{L1,\max} F_s} \tag{35}$$

$$L_2 = \frac{(1 - D_2)V_o}{2F_S I_{o,\min}}$$
(36)

Where, V_{ip} is the peak value of V_i , $\Delta I_{L1,max}$ is the maximum variation permited for L_1 current, and $I_{o,min}$ is the mínimum DC value of the output current.

The inductors built had the following measured values:

The capacitors used were:

$$C_1 = 200 \mu F$$
 $C_2 = 100 \mu F$.

VI. CONTROL SYSTEM

An algebraic study on the State-Equations was made and a control system was projected with standard PI (proportional plus integral) controlers, as indicated in fig. (3). The input control system uses a multiplier control technique [3], with a fast input current loop (PI_i), and a slow voltage loop (PI_1). The output is controled by a single voltage control loop (PI_o).

The values of proportional and integral constants calculated were applied to the Capacitive Idling Cuk Converter and to the Boost-Buck set as well.

VII. SIMULATION RESULTS

Some simulations were made in order to verify the converters behavior under full load operation and load variations. Fig. 4 presents results for the Capacitive Idling Cuk converter under load variations. Initially the converter operates under full load, then it operates under reduced load (¹/₄ full load) for a small amount of time, and returns to full load.



Fig. 3 - Control system applied to Capacitive Idling Cuk converter





It can be seen that the input current presents a nearly sinusoidal shape, the capacitor C_1 voltage (V_1) do not vary more than 5%, the output remains in its reference (69V), and duty ratio D_1 is mantained greater than D_2 all the time. The small high frequency oscilation in output voltage, when the load is increased, can be avoided placing an inner current (I_2) control loop.

The Boost-Buck set presented the same behavior as those presented for Capacitive Idling Cuk Converter. Simulation results with Boost-Forward set presented equality in performance to Boost-Buck, but it was not true for the Capacitive Idling Cuk converter with isolation.

VIII. EXPERIMENTAL RESULTS

It was buil low power (138W) prototypes for three of the converters, with the same characteristics and operating under equality of conditions. Fig. 5 presents the output voltage and current for the converters under load variation, from ¹/₄ full load to full load.







- - -

Fig. 6 - Output voltage (1) and current (2) - full-load to $\frac{1}{4}$ full-load

Fig. 6 presents the output voltage and current for the converters under load variation, from full load to ¹/₄ full load. The bad shape of the current was due to imprefections of the switch that changed the load.

In all the cases it was observed good output voltage regulation, even under hard load variation. The maximum rypple was $\pm 1V$ for Boost-Buck set and $\pm 1.5V$ for Capacitive Idling Cuk converter. Under load variation, the voltage fluctuation was no more than 4V.

Fig. 7 presents input voltage and current for the converters under full load operation. Current is the inner curve.



Fig. 7 - Input voltage (1) and current (2) under full-load

In percentual terms, only the 5^{th} harmonic component (9.7%) stayed above the percentual value related to Satandard IEC 61000-3-2 (7.1%), but it was due to the 5^{th} harmonic component present in the local AC line voltage (5%). Besides that, only the 3^{rd} harmonic component presented a significant value (8.2%).

In order to verify the viability in applying the results of the studies made for the Boost-Buck set to its isolated version, a Boost-Forward set was built with the same characteristics of Boost-Buck set, changing the output voltage specification to Vo=13.8V. The values of output capacitor, inductor, and proportional and integral constants was changed proportionally to the new value of output voltage. It was used an input LC (inductor plus capacitor) filter. The results obtained can be seen in fig. 8.

The measurement of the input voltage and current gave the following results of Power Factor (PF), Total Harmonic Distortion (THD) and efficiency (η) :

Converter	FP	THD	η
Boost-Buck set	0,95	13,8%	91%
Capacitive Idling Cuk converter	0,96	12,6%	81%
Boost-forward set	0,99	12,5%	80%



(c) Input voltage and current

Fig. 8 - Voltages (1) and currents (2) for the Boost-Forward set

IX. CONCLUSIONS

The experimental results confirmed The theoretical studies, validating the possibility of mading the control project separated for the two stages (pre-regulator and regulator).

The results with input current and output voltage indicated a good performance of the converters in respect to the desired parameters.

This work demonstrated the characteristics and performance similarity between the two converters studied. In comparative aspects, one can not say that there are signifacative differences between the two converters studied, but the efficiency obtained for the Capacitive Idling Cuk converter was a little lower than the efficiency obtained for the Boost-Buck set.

Regarding assembling tasks, the Capacitive Idling Cuk converter presents some characteristics distinct of the Boost-Buck set studied, namely, interleaving between S_1 and $S_2\,$, and the impracticability in conecting both switches in the same referencial point.

It was successfully experienced the possibility of applying the results of the studies and project of control to the isolated version of the Boost-Buck set with no practical prejudice in its performance. This was confirmed by the similaraties of the experimental results, obtained for this set and the Boost-Forward set.

X. References

- L.D. Stevanovic and S. Cuk, "Capacitive Idling Converters with Decoupled Input Voltage and Load Regulation Loops", *IEEE-PESC'93 Conference Record*, 1993, pp. 681-688.
- [2] Vendrusculo, E.A. & Pomilio, J.A.; "Conversores com Capacitor Flutuante e Comutação Suave: Aplicações em Fonte de Tensão com Alto Fator de Potência e Carga Eletrônica Regenerativa", *Eletrônica de Potência*, SOBRAEP, vol.2, no. 1, pp. 13-22, jun. 1997
- [3] J. Sebatián et alli, "Corrección del Factor de Potencia en Sistemas de Alimentación Monofásicos", SOBRAEP-COBEP'97 Conference Record, 1997, pp. 14-28.
- [4] C. A. Canesin and I. Barbi, "Pré-reguladores Retificadores de Elevado Fator de Potência para Sistemas de Telecomunicações, uma Visão Comparativa", SOBRAEP-COBEP'97 Conference Record, 1997, pp. 218-222
- [5] O. Garcia et alli, "A New Family of Single Stage AC/DC Power Factor Correction Converters with Fast Output Voltage Regulation", IEEE-PESC'97 Conference Record, 1997, pp. 536-542.
- [6] L.F.P.Mello, Análise e Projeto de Fontes Chaveadas. São Paulo, SP: Editora Érica Ltda, 1996, pp. 1-281
- [7] N. Mohan, Power Electronics: converters, applications and design. New York, NY: John Wiley & Sons, Inc., 1995, pp. 301-351.
- [8] I.Barbi and A. F. Souza, "Correção de Fator de Potência de Fontes de Alimentação - Curso", UFSC, Florianópolis, 1993.
- [9] J.C. Giacomin, "Estudo de Conversores Chaveados com Alto Fator de Potência na Entrada e Tensão Constante na Saída", *Dissertação de Mestrado*, UFMG, Belo Horizonte, 1998.